TIE-19 Temperature Coefficient of the Refractive Index

Introduction

The refractive index of optical glasses changes with temperature, the extend of which depending on the glass type and on the wavelength. It also changes with air pressure, therefore in the following we distinguish between the "refractive index relative to vacuum" or "absolute refractive index" and the refractive index at normal air pressure, called "refractive index relative to normal air" (relative refractive index, all values given in the Pocket Catalog are relative refractive index values). The standard measurement temperature for refractive index values given on test certificates is 22 °C [1]. By taking into account the temperature coefficients of the glasses it is possible to calculate the change of refractive index for any wavelength from near UV to near IR in a temperature range from −100 °C up to +140 °C. The constants of the formula are listed in the data sheets of the respective optical glass [2]. The calculation of the actual refractive index at a given temperature often seems not to be straight forward and the Technical Information TIE-29 [3] displays only the formula for the temperature coefficients for

the absolute refractive index. This Technical Information will give a guideline how to calculate the refractive index at different temperatures. Additionally some results of temperature coefficients for optical glasses will be given and discussed. Some special glasses with negative temperature coefficients in an optical system can help to keep wave front deformations caused by temperature changes on a minimum level. Such glasses are called athermal glasses.

1. Fundamentals

The refractive index of glasses relative to vacuum (absolute refractive index) can be described with sufficient approximation as a function of wavelength from the near IR through the visible to the near UV spectral region by a Sellmeier-type formula [4].

$$n^2_{abs}(\lambda) - 1 = a \cdot \frac{N}{V} \cdot f \cdot \frac{\lambda^2 \cdot \lambda^2_0}{\lambda^2 - \lambda^2_0} \qquad (1)$$

with:

$n_{abs}(\lambda)$ refractive index relative to vacuum
λ wavelength of the electromagnetic wave in vacuum
a constant
N average number of oscillators in volume V
f average oscillator strength
λ_0 average resonant wavelength of the electronic oscillator in the UV

The product $a \cdot \frac{N}{V} \cdot f$ as well as λ_0 can be determined for a chosen temperature (for example room temperature) by fitting the measured data for $n_{abs}(\lambda)$ at this temperature. This parameter is temperature independent.

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2. Derivation and Characterization of the Formula

The derivative of the Sellmeier function (1) with temperature yields equation (2) after simplification by omitting terms with negligible contributions. The coefficients D_0, D_1, D_2, E_0, E_1 and λ_{TK} for a given glass type will be determined by fitting the experimentally obtained data [4].

$$\frac{dn_{abs}(\lambda, T)}{dT} = \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot$$
$$\left(D_0 + 2 \cdot D_1 \cdot \Delta T + 3 \cdot D_2 \cdot \Delta T^2 + \frac{E_0 + 2 \cdot E_1 \cdot \Delta T}{\lambda^2 - \lambda_{TK}^2}\right) \qquad (2)$$

with:

T_0 Reference temperature (20 °C)
T Temperature (in °C)
ΔT Temperature difference $T-T_0$
λ Wavelength of the electromagnetic wave in a vacuum (in µm)
D_0, D_1, D_2, E_0, E_1 and λ_{TK}: constants depending on glass type

The refractive index values given in the Optical Glass Pocket Catalog apply for an air pressure of $0.10133 \cdot 10^6$ Pa. They are called relative refractive index values n_{rel}. These values can be used for $n(\lambda, T_0)$ in equation (2) with sufficient accuracy.

When the function of temperature is closed integrable, the change in the absolute refractive index $n_{abs}(\lambda, T_0)$, with the temperature difference ΔT can be derived from equation (2):

$$\Delta n_{abs}(\lambda, T) = \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot$$
$$\left(D_0 \cdot \Delta T + D_1 \cdot \Delta T^2 + D_2 \cdot \Delta T^3 + \frac{E_0 \cdot \Delta T + E_1 \cdot \Delta T^2}{\lambda^2 - \lambda_{TK}^2}\right) \qquad (3)$$

and

$$n_{abs}(\lambda, T) = n_{abs}(\lambda, T_0) + \Delta n_{abs}(\lambda, T) \qquad (4)$$

Consequently, two simple closed expressions for the calculation of the temperature coefficient of the absolute refractive index: $\frac{dn_{abs}(\lambda, T)}{dT}$, as well as the change of the absolute refractive index with temperature: $\Delta n_{abs}(\lambda, T)$ are available.

The refractive index relative to air $\frac{dn_{rel}(\lambda, T)}{dT}$ and $\Delta n_{rel}(\lambda, T)$ can be calculated from:

$$n_{rel}(\lambda, T) = \frac{n_{abs}(\lambda, T)}{n_{air}(\lambda, T, p)} \qquad (5)$$

and

$$\frac{dn_{abs}(\lambda, T)}{dT} = n_{air}(\lambda, T, p) \cdot \frac{dn_{rel}(\lambda, T)}{dT} +$$
$$n_{rel}(\lambda, T) \cdot \frac{dn_{air}(\lambda, T, p)}{dT} \qquad (6)$$

or

$$\frac{dn_{rel}(\lambda, T)}{dT} = \frac{\frac{dn_{abs}(\lambda, T)}{dT} - n_{rel}(\lambda, T) \cdot \frac{dn_{air}(\lambda, T, p)}{dT}}{n_{air}(\lambda, T, p)} \qquad (7)$$

In (6) and (7) $n_{rel}(\lambda, T_0)$ can be substituted for $n_{rel}(\lambda, T)$ with sufficient accuracy.

The value for $n_{air}(\lambda, T, p)$ and $\frac{dn_{air}(\lambda, T, p)}{dT}$ can be calculated with good accuracy with [4]

$$n_{air}(\lambda, T, P) = 1 + \frac{n_{air}(\lambda, 15\,°C, P_0) - 1}{\left(1 + 3.4785 \cdot 10^{-3} \frac{1}{°C} \cdot (T - 15\,°C)\right)} \cdot \frac{P}{P_0} \qquad (8)$$

$$n_{air}(\lambda, 15\,°C, P_0) = 1 + 10^{-8} \cdot$$
$$\left(6432.8 + \frac{2949810 \frac{1}{µm^2} \cdot \lambda^2}{\left(146 \frac{1}{µm^2} \cdot \lambda^2 - 1\right)} + \frac{25540 \frac{1}{µm^2} \cdot \lambda^2}{\left(41 \frac{1}{µm^2} \cdot \lambda^2 - 1\right)}\right) \qquad (9)$$

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$$\frac{dn_{air}(\lambda, T, P)}{dT} = -0.00367 \cdot \frac{n_{air}(\lambda, T, P) - 1}{1 + 0.00367 \frac{1}{^\circ C} \cdot T} \tag{10}$$

with:

P_0 $0.101325 \cdot 10^6$ Pa (normal pressure in Pascal)
P Air pressure
λ Wavelength of the electromagnetic wave in
     a vacuum (in µm)
T Temperature (in °C)

With the help of 6 glass specific parameters D_0, D_1, D_2, E_0, E_1 and λ_{TK} the value of the temperature coefficient of the absolute refractive index (compared to vacuum) as a function of temperature and wavelength can be calculated with equation (2) for each glass type. The variation of the absolute refractive

index compared to the value at 20 °C can be calculated with equation (3). The corresponding value for air can be calculated by (5) and (7).

The constants of formula (2) and (3) are given in the glass data sheets [2]. The constants are valid for a temperature range from −100 °C to +140 °C and a wavelength range from 0.3650 µm to 1.014 µm. The temperature coefficients in the data sheets are guideline values.

Upon request, measurements can be performed on individual melts for the temperature and wavelength range as given above with a precision better than ± $5 \cdot 10^{-7}$/K. The constants of the dispersion formula can be calculated from this measured data and will be listed on the test certificate for the individual glass melt on request.

3. Temperature Coefficients of Optical Glasses

The extent of change of the refractive index at a given wavelength varies from glass to glass. Table 1 shows the list of our current product range sorted by the relative temperature coef-

ficients between +20 °C and +40 °C at the e-line (~546.1 nm). The values reach from +24.1 $\cdot 10^{-6}$/K for P-SF68 down to $-6.7 \cdot 10^{-6}$/K for N-PK51.

Glass	n_d	v_d	Relative temperature coefficients of refractive index [+20/+40 deg. °C]		
			1060 nm	e-line	g-line
P-SF68	2.00520	21.00	15.20	24.10	36.50
SF11	1.78472	25.76	9.20	12.90	17.60
SF57	1.84666	23.83	7.60	12.50	18.90
SF57HTultra	1.84666	23.83	7.60	12.50	18.90
SF6	1.80518	25.43	6.80	11.10	16.20
SF6HT	1.80518	25.43	6.80	11.10	16.20
SF56A	1.78470	26.08	6.20	10.00	14.70
N-LASF40	1.83404	37.30	7.30	9.30	11.40
SF4	1.75520	27.58	5.70	9.20	13.30
P-LASF50	1.80860	40.46	7.10	8.90	10.60
N-LASF46B	1.90366	31.32	6.40	8.90	11.80
P-LASF51	1.81000	40.93	6.90	8.70	10.40
P-LASF47	1.80610	40.90	6.90	8.60	10.30
N-LAF33	1.78582	44.05	7.00	8.50	10.00

Glass	n_d	v_d	Relative temperature coefficients of refractive index [+20/+40 deg. °C]		
			1060 nm	e-line	g-line
N-LAF35	1.74330	49.40	7.10	8.40	9.60
LAFN7	1.74950	34.95	6.30	8.30	10.40
SF10	1.72825	28.41	5.30	8.10	11.60
SF1	1.71736	29.51	5.00	7.90	11.30
N-ZK7	1.50847	61.19	6.40	7.00	7.60
N-LASF46A	1.90366	31.32	4.70	7.00	9.80
N-LASF43	1.80610	40.61	5.00	6.50	8.10
SF5	1.67270	32.21	3.50	5.80	8.40
N-LASF45	1.80107	34.97	3.80	5.70	7.90
N-LASF45HT	1.80107	34.97	3.80	5.70	7.90

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Tab. 1: Product range sorted by relative temperature coefficients between +20 °C and +40 °C at the e-line.

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Glass	n_d	v_d	Relative temperature coefficients of refractive index [+20/+40 deg. °C]		
			1060 nm	e-line	g-line
N-KZFS5	1.65412	39.70	4.20	5.50	6.80
LASF35	2.02204	29.06	2.70	5.50	9.00
N-LAF34	1.77250	49.62	4.30	5.40	6.50
N-LASF41	1.83501	43.13	4.00	5.40	6.80
N-KZFS2	1.55836	54.01	4.70	5.30	5.90
N-LASF44	1.80420	46.50	4.00	5.30	6.50
N-SSK2	1.62229	53.27	4.30	5.20	6.10
N-LAF21	1.78800	47.49	3.90	5.10	6.20
N-LAK10	1.72003	50.62	4.20	5.10	6.10
N-BALF4	1.57956	53.87	4.20	5.10	6.00
N-LASF9	1.85025	32.17	2.90	5.10	7.70
N-LASF9HT	1.85025	32.17	2.90	5.10	7.70
N-LAK8	1.71300	53.83	4.10	5.00	5.80
N-LASF31A	1.88300	40.76	3.30	4.90	6.60
N-BAF10	1.67003	47.11	3.80	4.90	6.00
N-KZFS11	1.63775	42.41	3.50	4.60	5.70
F5	1.60342	38.03	3.00	4.60	6.20
SF2	1.64769	33.85	2.70	4.60	6.90
N-LAK33B	1.75500	52.30	3.50	4.50	5.40
N-SK2	1.60738	56.65	3.60	4.50	5.30
N-SK2HT	1.60738	56.65	3.60	4.50	5.30
F2	1.62004	36.37	2.70	4.40	6.30
F2HT	1.62004	36.37	2.70	4.40	6.30
N-BASF2	1.66446	36.00	2.90	4.40	6.20
N-BASF64	1.70400	39.38	2.80	4.30	5.90
N-LAF7	1.74950	34.82	2.60	4.30	6.30
K10	1.50137	56.41	3.60	4.20	4.90
N-LAK14	1.69680	55.41	3.20	4.00	4.70
N-KZFS8	1.72047	34.70	2.40	4.00	5.80
N-SF2	1.64769	33.82	2.60	4.00	5.90
N-BAK4	1.56883	55.98	3.10	3.90	4.70
N-BAK4HT	1.56883	55.98	3.10	3.90	4.70
N-LAK34	1.72916	54.50	3.00	3.80	4.60
P-SK58A	1.58913	61.15	3.20	3.80	4.40

Glass	n_d	v_d	Relative temperature coefficients of refractive index [+20/+40 deg. °C]		
			1060 nm	e-line	g-line
N-KZFS4	1.61336	44.49	2.70	3.70	4.70
N-KZFS4HT	1.61336	44.49	2.70	3.70	4.70
N-LAK9	1.69100	54.71	2.90	3.70	4.40
N-SK5	1.58913	61.27	3.20	3.70	4.30
P-SK60	1.61035	57.90	2.90	3.60	4.30
P-SK57	1.58700	59.60	2.90	3.60	4.30
N-F2	1.62005	36.43	2.10	3.50	5.10
N-BK10	1.49782	66.95	2.90	3.40	3.80
N-SF5	1.67271	32.25	1.80	3.40	5.50
N-SF15	1.69892	30.20	1.60	3.40	5.80
N-BAF52	1.60863	46.60	2.30	3.30	4.30
N-BAF4	1.60568	43.72	2.20	3.30	4.50
N-SSK5	1.65844	50.88	2.20	3.20	4.20
N-SK11	1.56384	60.80	2.60	3.20	3.80
N-LAK22	1.65113	55.89	2.40	3.10	3.90
N-SK14	1.60311	60.60	2.40	3.10	3.70
SCHOTT N-BK7®	1.51680	64.17	2.40	3.00	3.50
N-BK7HT	1.51680	64.17	2.40	3.00	3.50
N-PSK3	1.55232	63.46	2.50	3.00	3.50
N-BAF51	1.65224	44.96	1.70	2.90	4.10
LLF1	1.54814	45.75	1.90	2.90	3.90
N-BALF5	1.54739	53.63	2.10	2.90	3.70
N-SSK8	1.61773	49.83	2.00	2.90	3.90
N-SK4	1.61272	58.63	2.10	2.80	3.40
N-SF8	1.68894	31.31	0.90	2.60	4.80
N-BAK1	1.57250	57.55	1.80	2.50	3.20
N-SF11	1.78472	25.68	0.10	2.40	5.60
N-SF66	1.92286	20.88	−0.50	2.40	7.30
N-SK16	1.62041	60.32	1.70	2.30	2.90
N-SF57	1.84666	23.78	−0.50	2.20	6.00
N-SF57HT	1.84666	23.78	−0.50	2.20	6.00
N-SF57HTultra	1.84666	23.78	−0.50	2.20	6.00

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Tab. 1: Product range sorted by relative temperature coefficients between +20 °C and +40 °C at the e-line.

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Glass	n_d	v_d	Relative temperature coefficients of refractive index [+20/+40 deg. °C]		
			1060 nm	e-line	g-line
N-K5	1.52249	59.48	1.40	2.10	2.70
LF5HTi	1.58144	40.89	0.80	2.00	3.40
LF5	1.58144	40.85	0.80	2.00	3.40
N-KF9	1.52346	51.54	0.90	1.80	2.60
N-SF1	1.71736	29.62	0.00	1.80	4.20
P-LAK35	1.69350	53.20	0.80	1.70	2.60
N-BAK2	1.53996	59.71	1.00	1.70	2.30
K7	1.51112	60.41	0.90	1.60	2.20
P-SF8	1.68893	31.25	−0.30	1.50	3.70
N-SF10	1.72828	28.53	−0.50	1.50	4.10
N-SF6	1.80518	25.36	−0.80	1.50	4.80
N-SF6HT	1.80518	25.36	−0.80	1.50	4.80
N-SF6HTultra	1.80518	25.36	−0.80	1.50	4.80
N-SF4	1.75513	27.38	−0.70	1.40	4.20
N-SF14	1.76182	26.53	−1.10	1.10	4.10
N-LAF2	1.74397	44.85	−0.10	1.00	2.30
N-LAK21	1.64049	60.10	0.50	1.00	1.60
N-LAK7	1.65160	58.52	0.00	0.70	1.30
N-LAK12	1.67790	55.20	−1.20	−0.40	0.30
N-FK5	1.48749	70.41	−1.40	−1.00	−0.60
N-PSK53A	1.61800	63.39	−2.90	−2.40	−1.80
N-FK51A	1.48656	84.47	−6.00	−5.70	−5.30
N-FK58	1.45600	90.90	−6.50	−6.20	−5.90
N-PK52A	1.49700	81.61	−6.70	−6.40	−6.00
N-PK51	1.52855	76.98	−7.10	−6.70	−6.40

Tab. 1: List of our current product range sorted by the relative temperature coefficients between +20 °C and +40 °C at the e-line (~546.1 nm).

Figures 1 to 4 show the change of the temperature coefficients of refractive index (absolute on the left side and relative on the right side) with temperature for different wavelengths and different glasses. It can be seen that the shape of the curve of the relative temperature coefficients differs from the absolute temperature coefficients. The reason is the influence of the temperature coefficients of the air. In general the influence of the temperature on the refractive index at shorter wavelengths is much higher compared to longer wavelengths. This "dispersion" of the temperature coefficient is more pronounced for high dispersion glasses.

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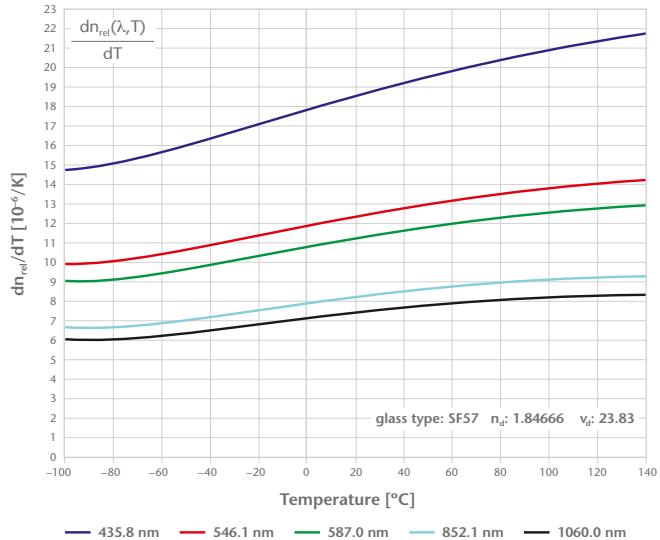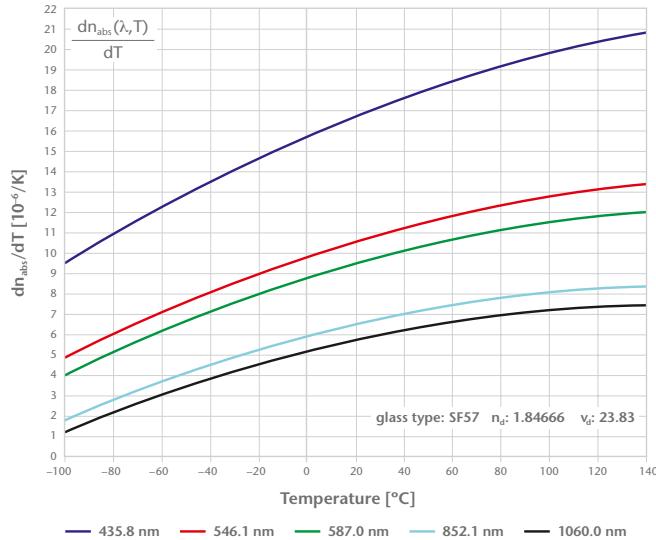
Fig. 1: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of **SF57** for different wavelengths.
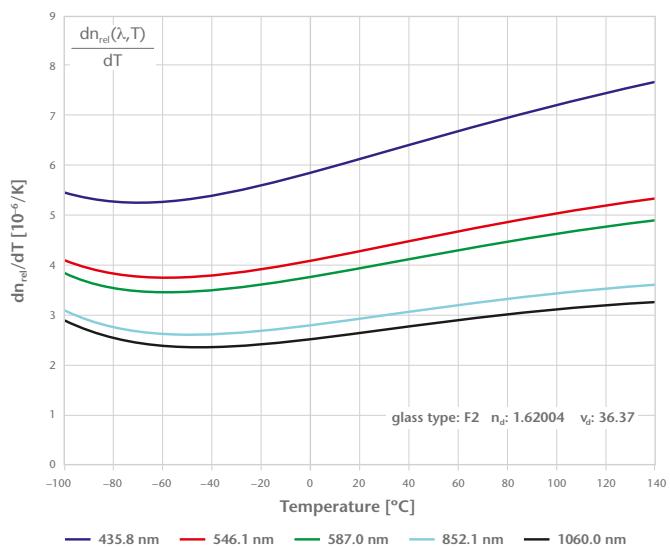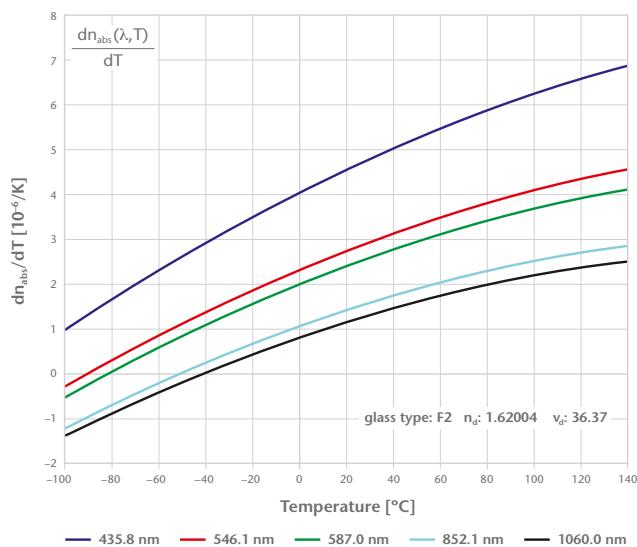

**Fig. 2:** Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of **F2** for different wavelengths.

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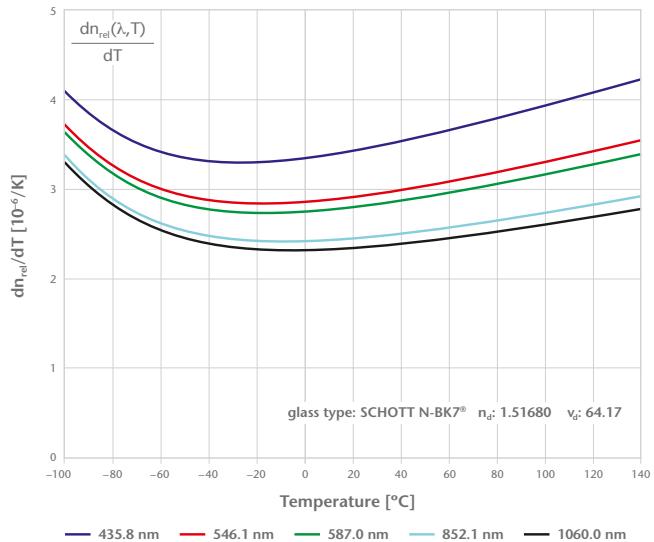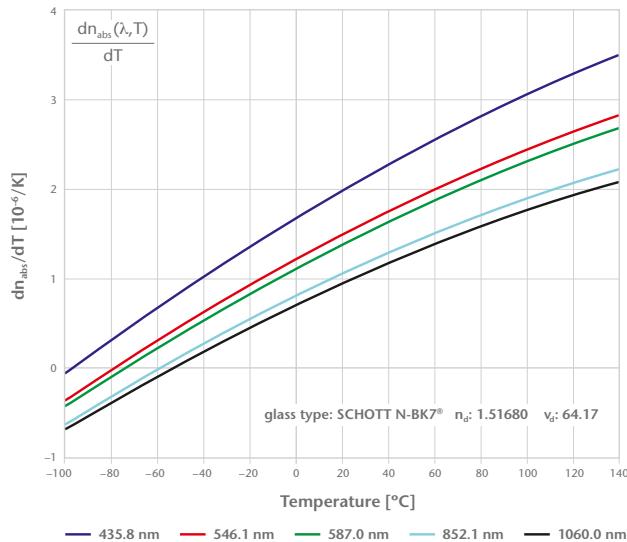
Fig. 3: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of **SCHOTT N-BK7®** for different wavelengths.
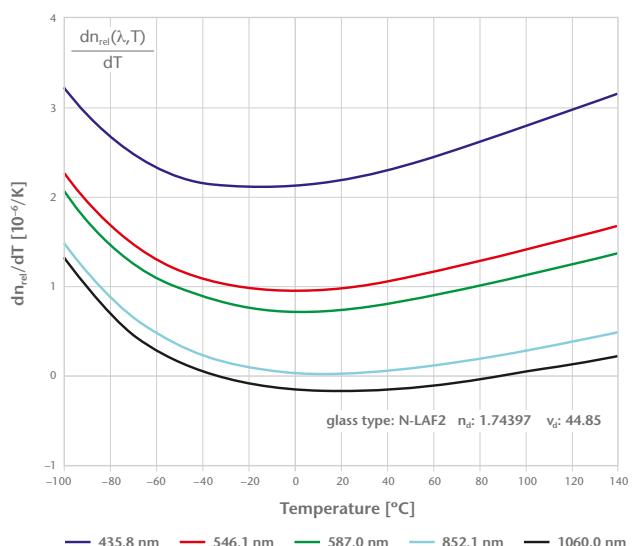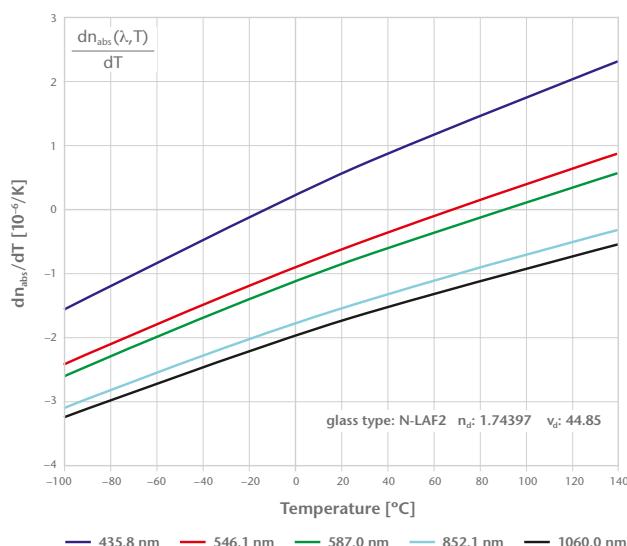
Fig. 4: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of **N-LAF2** for different wavelengths.

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Figure 5 summarizes the above displayed examples in one diagram to better pronounce the different temperature coefficients of different glasses. Often it is easier to display the absolute values of refractive index as a function of temperature. In figure 6 the change of the relative refractive index with temperature is displayed for the above discussed glass types.
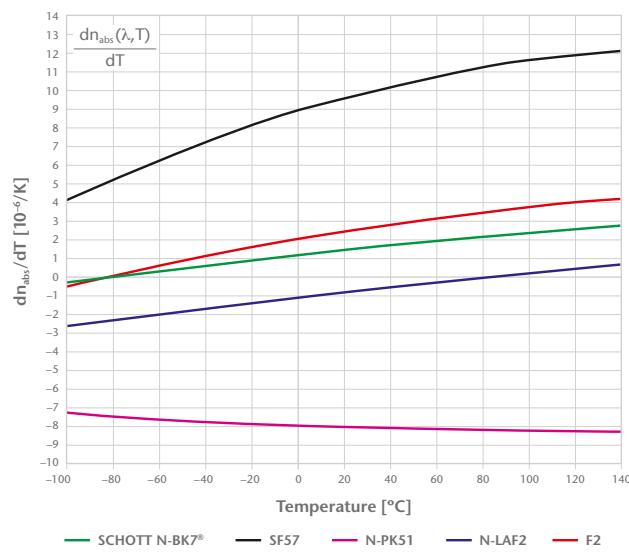
It can be seen that a temperature increase of 20 °C changes the refractive index of SF57 at 587 nm by $2.3 \cdot 10^{-4}$. Additionally the diagrams also contain the temperature change curves of N-PK51. This glass has a negative temperature coefficient over the complete temperature range.
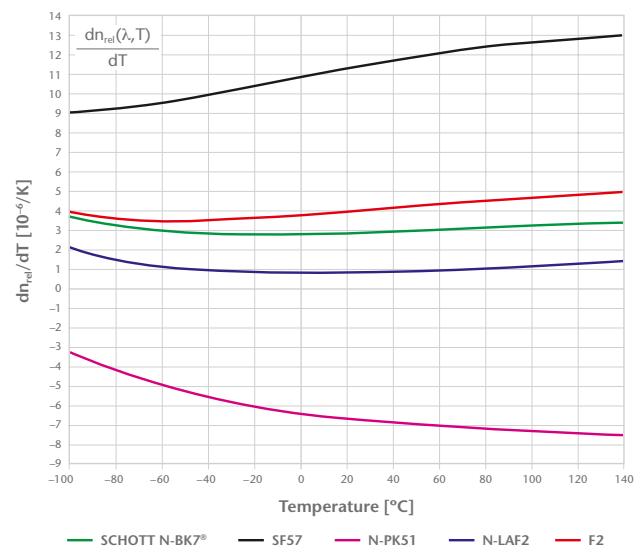
Fig. 5: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of different glass types in comparison at 587 nm.
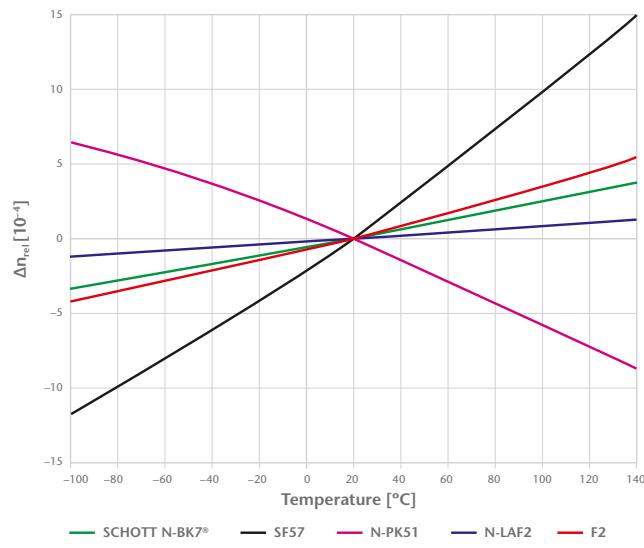
Fig. 6: Change of the relative refractive index (catalog value at 20 °C) with temperature of some different glass types at 587 nm.

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4. Athermal Glasses

In conditions, where the optical system is subjected to varying inhomogeneous temperature environments, wave front distortions can be expected due to changes in optical path length and refractive index with temperature.

To compensate such effects, it is important that the effect of wave front distortions generated due to the inhomogeneous thermal expansion of the glass can be compensated by the refractive index change with temperature. The fact that the thermal expansion coefficient of optical glasses is always positive implies the need of optical glasses with a negative temperature coefficient to compensate thermal effect on the wave front. This can be expressed in the following equation:

$$G = \alpha \cdot (n_{rel}(\lambda, T) - 1) + \frac{dn_{rel}(\lambda, T)}{dT} \qquad (10)$$

α is the thermal expansion coefficient of the glass, $n_{rel}(\lambda, T)$ the refractive index at the proposed wavelength. The value thermo optical constant G should be as low as possible. Glasses with a value of G near zero are called athermal glasses. Table 2 shows that not all glasses exhibiting a negative temperature coefficient are excellent athermal glasses.

Optical glasses with negative temperature coefficients are N-LAK12, N-PSK53A, N-FK5, N-FK51A, N-PK52A, N-FK58 and N-PK51. But only N-PK51, N-FK58, N-PK52A and N-FK51A show a real athermal behavior.

Glass	n_e	Relative temperature coefficients of refractive index [+20/+40 deg. °C], e-Line [10^{-6}K]	alpha −30/70 [10^{-6}K]	G [10^{-6}K]
N-PK51	1.53019	−6.70	12.35	−0.15
N-FK58	1.45600	−6.20	13.67	0.03
N-PK52A	1.49845	−6.40	13.01	0.08
N-FK51A	1.48794	−5.70	12.74	0.52
N-FK5	1.48914	−1.00	9.2	3.50
N-PSK53A	1.62033	−2.40	9.56	3.53
N-LAK12	1.68083	−0.40	7.6	4.77
SCHOTT N-BK7®	1.51872	3.00	7.1	6.68
N-LAF2	1.74791	1.00	8.06	7.03
F2	1.62408	4.40	8.2	9.52
SF57	1.85504	12.50	8.3	19.60

Tab. 2: Optical glasses and their thermo optical constant G.

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TIE-19 Temperature Coefficient of the Refractive Index

Figure 7 shows the temperature coefficient of N-PK51. It can be seen that the values are negative over the complete temperature region from −100 °C to +140 °C.
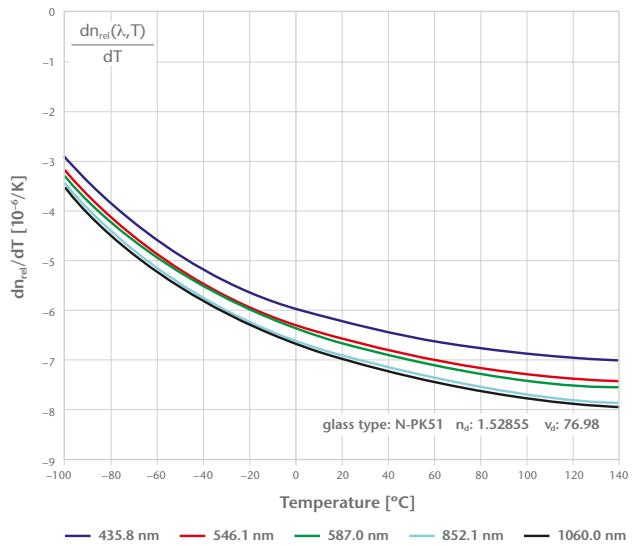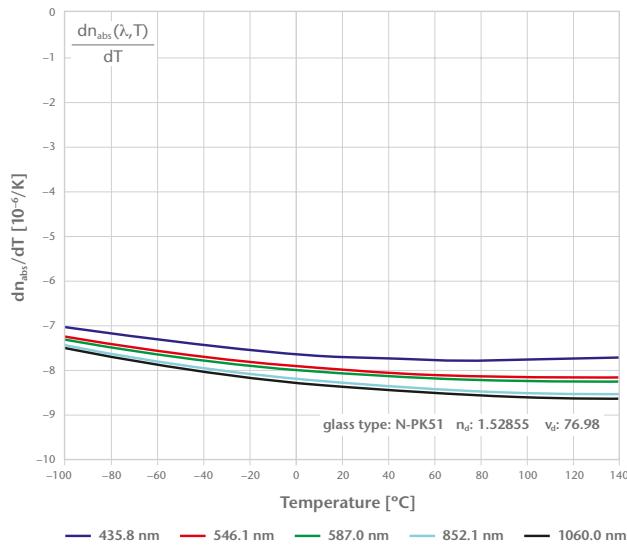

Fig. 7: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of N-PK51 for different wavelengths.

5. Literature

[1] SCHOTT Optical Glass Pocket Catalog 2016

[2] Data Sheets Optical Glasses from SCHOTT

[3] SCHOTT Technical Information TIE-29 "Refractive index and dispersion"

[4] "The properties of optical glass", Bach, Neuroth (Eds.), Springer 1998

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